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# Estimation of MIMO Channel Capacity from Phase-Noise Impaired Measurements

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**Abstract**—Due to the significantly reduced cost and effort for system calibration time-division multiplexing (TDM) is a commonly used technique to switch between the transmit and receive antennas in multiple-input multiple-output (MIMO) radio channel sounding. Nonetheless, Baum *et al.* [1], [2] have shown that phase noise of the transmitter and receiver local oscillators, when it is assumed to be a white Gaussian random process, can cause large errors of the estimated channel capacity of a low-rank MIMO channel when the standard channel matrix estimator is used. Experimental evidence shows that consecutive phase noise samples affecting measurement samples collected with real TDM-MIMO channel sounders are correlated. In this contribution a capacity estimator that accounts for the phase noise correlation is proposed. The estimator is based on a linear minimum mean square error estimate of the MIMO channel matrix. It is shown by means of Monte Carlo simulations assuming a measurement-based phase noise model, that the MIMO channel capacity can be estimated accurately for signal-to-noise ratios up to about 35 dB.

## I. INTRODUCTION

To save hardware cost and alleviate the needed calibration procedures, most advanced multiple-input multiple-output (MIMO) radio channel sounders rely on a time-division multiplexing (TDM) technique. In such a system, which is represented schematically in Fig. 1, a single sounding waveform generator is connected to a number of transmit antennas via a switch. Similarly, the output terminals of the receive array are sensed via another switch. Thereby channel observations are made via a spatio-temporal aperture [3].

It has been shown recently that concatenated phase noise of the two oscillators in the transmitter and the receiver affects the estimation of MIMO channel capacity when using the standard channel matrix estimator to obtain a capacity estimate [1], [4]. For short we call this concatenated noise the phase noise of the sounding system. The effect of phase noise on MIMO capacity estimation is studied in [4] assuming that phase noise is a random walk process. Theoretical investigations reported in [1], [2] show that, provided phase noise is white

and Gaussian, it leads to large measurement errors in terms of estimated channel capacity of a low-rank MIMO channel. In [2] analytical results are given under the assumptions that the TDM, i.e. the spatio-temporal array [3], fulfills a separability condition and that the phase noise process is white. However, experimental studies reported in [5] show that phase noise cannot be assumed to be white or a random walk on the time-scale of a measurement period [6], [5]. In addition, the spatio-temporal array induced by the used switching schemes [3] determines the ordering of the phase noise samples in the estimation of the standard channel matrix estimate. Both effects significantly affect the performance of capacity estimation based on this matrix estimator [7].

In this paper we propose a new method for estimation of the channel capacity from phase-noise impaired measurement data. The estimator relies on linear minimum mean-square-error (MMSE) estimation of the channel transfer matrix. The performance of the proposed estimator in terms of estimation accuracy is compared to standard estimators using the phase-noise model developed in [5].

## II. SIGNAL MODEL FOR PHASE-NOISE IMPAIRED TDM-MIMO SOUNDING

We consider the TDM sounding system depicted schematically in Fig. 1 with  $N$  transmit antennas and  $M$  receive antennas. To allow for measurements of the full  $M \times N$  channel matrix  $\mathbf{H}$ ,  $[\mathbf{H}]_{mn} = h_{mn}$ , the sounder is equipped with a switch at the transmitter and a switch at the receiver. The channel matrix  $\mathbf{H}$  is assumed to be constant during one measurement run.<sup>1</sup> The coefficient  $h_{mn}$  of the sub-channel consisting of the  $n$ th transmit array element, the propagation channel, and the  $m$ th receive array element is measured with the transmitter switch in position  $n$  and the receiver switch in position  $m$  (see Fig. 1). The receiver acquires  $K$  samples indexed by  $k = 1, \dots, K$ . Sample  $k$  is obtained at time  $t_k$  with the transmitter switch in position  $n(k) \in \{1, \dots, N\}$  and receiver switch in position  $m(k) \in \{1, \dots, M\}$ . Thus, at time instant  $t_k$  the system performs a measurement of the channel coefficient  $h_{m(k)n(k)}$ . The sequence  $\{(t_k, m(k), n(k))\}$  defines the spatio-temporal array of the sounding system [8], [3]. We define the index set  $\mathcal{K}_{mn}$  to be the set of sample

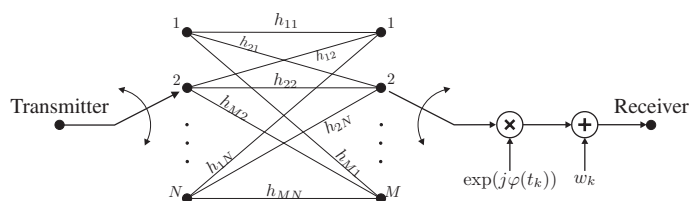


Fig. 1. Model for TDM-MIMO channel sounding with phase noise.

<sup>1</sup>The validity of this assumption depends on how rapidly the channel varies and on the duration of the measurement. Assuming a stationary channel is necessary for the definition of channel capacity given in Subsection III-A.

indices for which sub-channel  $(m, n)$  is measured, i.e.

$$\mathcal{K}_{mn} = \{k : (m(k), n(k)) = (m, n)\}. \quad (1)$$

The cardinality of  $\mathcal{K}_{mn}$ , denoted by  $\#\mathcal{K}_{mn}$ , is equal to the number of samples acquired from sub-channel  $(m, n)$ .

As depicted in Fig. 1, the observed signal is modulated with a time-varying phasor  $\exp(j\varphi(t_k))$  due to the phase noise in the oscillators of the sounding system. The  $k$ th sample is modeled as

$$g_k = h_{m(k)n(k)} \cdot \exp(j\varphi(t_k)) + w_k, \quad (2)$$

where  $\{w_k\}$  is a white Gaussian noise process with sample variance  $\sigma_w^2$ . We define the measurement signal-to-noise ratio (SNR) as  $\gamma = \sigma_h^2 / \sigma_w^2$  where  $\sigma_h^2$  is the variance of one channel coefficient. We consider the case where phase noise  $\varphi(t)$  can be modeled as a wide-sense stationary process with mean zero and a known autocorrelation function  $R_\varphi(\tau)$ . This assumption hold true if the time-span during which measurements are acquired is sufficiently short. With the time-span considered in the following, this condition can be met by commercially available channel sounders [5].

We define a  $K \times MN$  sounding matrix  $\mathbf{S}$  that rearranges the vectorized channel matrix  $\text{vec}(\mathbf{H})$  according to the order in which the sub-channels are measured:

$$\mathbf{S} \cdot \text{vec}(\mathbf{H}) = \begin{bmatrix} h_{m(1)n(1)} \\ \vdots \\ h_{m(k)n(k)} \\ \vdots \\ h_{m(K)n(K)} \end{bmatrix}, \quad (3)$$

i.e. the entries of row  $k$  of the sounding matrix are all zeros except for the entry corresponding to the entry  $h_{m(k)n(k)}$  of the vectorized channel matrix  $\text{vec}(\mathbf{H})$ . As an example consider a sounding system with  $M_1 = M_2 = 2$ , and  $K = 8$  using the identity sounding mode with  $[n(1), \dots, n(8)] = [1, 1, 2, 2, 1, 1, 2, 2]$ ,  $[m(1), \dots, m(8)] = [1, 2, 1, 2, 1, 2, 1, 2]$ . In this case,

$$\mathbf{S} \cdot \text{vec}(\mathbf{H}) = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & 1 & & \\ & & & 1 \\ & & 1 & \\ & & & 1 \\ & & & & 1 \end{bmatrix} \cdot \begin{bmatrix} h_{11} \\ h_{21} \\ h_{12} \\ h_{22} \end{bmatrix} = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{12} \\ h_{22} \\ h_{11} \\ h_{21} \\ h_{12} \\ h_{22} \end{bmatrix}. \quad (4)$$

We can now, after defining the three vectors

$$\mathbf{g} \triangleq \begin{bmatrix} g_1 \\ \vdots \\ g_K \end{bmatrix}, \quad \boldsymbol{\varphi} \triangleq \begin{bmatrix} \varphi_1 \\ \vdots \\ \varphi_K \end{bmatrix}, \quad \text{and} \quad \mathbf{w} \triangleq \begin{bmatrix} w_1 \\ \vdots \\ w_K \end{bmatrix}, \quad (5)$$

recast (2) in the compact form

$$\mathbf{g} = [\mathbf{S} \cdot \text{vec}(\mathbf{H})] \circ \exp(j\boldsymbol{\varphi}) + \mathbf{w}, \quad (6)$$

where  $\circ$  denotes the Hadamard (or element-wise) product and the exponential function is taken element-wise. Notice that if

$t_1 < t_2 < \dots < t_K$  then the entries of the vectors defined in (5) are ordered according to the temporal order.

In the case where each of the sub-channels is measured  $I$  times, the obtained measurements can be arranged in matrices  $\{\mathbf{G}_i\}$  such that  $[\mathbf{G}_i]_{m(k)n(k)} = g_k$  where  $i(k) \in \{1, \dots, I\}$  is a cycle-index assigned to sample  $k$ . There is a certain degree of freedom in the choice of  $i(k)$ : if the samples  $g_k$  and  $g_{k'}$  are both acquired from the same sub-channel we are free to choose to assign the cycle indices as  $i(k) = 1, i(k') = 2$  or  $i(k) = 2, i(k') = 1$ .

### III. ESTIMATION OF CAPACITY

When the channel is not known at the transmitter, but fully known at the receiver, its capacity at SNR  $\rho$  reads [9]

$$C(\mathbf{H}\mathbf{H}^H) = \log_2 \det(\mathbf{I}_M + \frac{\rho}{N} \mathbf{H}\mathbf{H}^H), \quad (7)$$

where  $\mathbf{H}^H$  denotes the Hermitian transpose of  $\mathbf{H}$ . The problem considered here is to estimate the capacity  $C(\mathbf{H}\mathbf{H}^H)$  from the noisy observation  $\mathbf{g}$ . It is important to distinguish between the SNR  $\rho$  in (7) at which we compute the capacity and the SNR  $\gamma$  during the measurement of  $\mathbf{g}$ . In general we wish to be able to compute capacities for other SNRs than the SNR prevailing during the measurement, i.e. for  $\rho \neq \gamma$ .

#### A. The Standard Capacity Estimator

The standard capacity estimator is defined as [1], [2], [7]

$$\hat{C}_{\text{std}} = C(\widehat{\mathbf{H}\mathbf{H}^H}), \quad \text{with} \quad \widehat{\mathbf{H}\mathbf{H}^H} = \frac{1}{I} \sum_{i=1}^I \mathbf{G}_i \mathbf{G}_i^H, \quad (8)$$

where  $\widehat{(\cdot)}$  denotes the estimate of the random element given as an argument.

We remark that the standard estimator can be applied only when  $I$  samples from each sub-channel are available. It is also worth mentioning that the standard estimator depends on the choice of  $i(k)$ . Therefore for the remainder of the paper we chose  $i(k)$  according to the temporal ordering of the samples, i.e. the first sample of sub-channel  $(m, n)$  is in  $\mathbf{G}_1$  and the second sample is in  $\mathbf{G}_2$ , etc.

#### B. Capacity Estimation by Averaging [6]

In [6] it is proposed to estimate the channel matrix by computing the average of  $\mathbf{H}$  of the data acquired during the measurement:

$$\hat{\mathbf{H}}_{\text{avg}} = \frac{1}{I} \sum_{i=1}^I \mathbf{G}_i. \quad (9)$$

This estimator can be generalized to non-cycled sounding as:

$$[\hat{\mathbf{H}}_{\text{avg}}]_{mn} = \frac{1}{\#\mathcal{K}_{mn}} \sum_{k \in \mathcal{K}_{mn}} g_k. \quad (10)$$

The capacity estimate  $\hat{C}_{\text{avg}}$  is then defined as  $\hat{C}_{\text{avg}} \triangleq C(\hat{\mathbf{H}}_{\text{avg}} \hat{\mathbf{H}}_{\text{avg}}^H)$ . This estimator leads to an estimation error lower than that of the standard estimator [6] and is independent of the choice of  $i(k)$ .

### C. Capacity Estimator based on a Linear MMSE Channel Estimate

Neither of the above estimators exploit the knowledge of the phase noise autocorrelation  $R_\varphi(\tau)$ . In the following we develop a new estimator for  $\mathbf{H}$  that takes this knowledge into account. The estimator relies on separate estimation of the moduli (magnitudes) and the arguments (phases) of the channel coefficients  $\{h_{mn}\}$ . Knowing the magnitude matrix  $\mathbf{Z} \triangleq |\mathbf{H}|$  and the phase angle matrix  $\mathbf{Y} \triangleq \angle \mathbf{H}$  we can recover the channel transfer matrix as

$$\mathbf{H} = \mathbf{Z} \circ \exp(j\mathbf{Y}). \quad (11)$$

Similarly, an estimate of  $\mathbf{H}$  can be obtained from estimates of  $\mathbf{Z}$  and  $\mathbf{Y}$  as

$$\hat{\mathbf{H}} = \hat{\mathbf{Z}} \circ \exp(j\hat{\mathbf{Y}}). \quad (12)$$

We estimate the magnitude matrix  $\mathbf{Z}$  by averaging the magnitudes of the acquired measurement data as

$$[\hat{\mathbf{Z}}]_{mn} = \frac{1}{\#\mathcal{K}_{mn}} \sum_{k \in \mathcal{K}_{mn}} |g_k|. \quad (13)$$

It is straightforward to show that  $[\hat{\mathbf{Z}}]_{m,n}$  this is an asymptotically consistent estimator of  $|h_{mn}|$  when the SNR  $\gamma$  tends to infinity.

The estimate of  $\mathbf{Y}$  is less obvious. Using the definition of the phase operator  $\angle_a$  provided in Appendix I, we define the vector  $\mathbf{x} \triangleq [x_1, \dots, x_K]^T$  of phases

$$x_k \triangleq \angle_{a_{m(k)n(k)}} g_k, \quad (14)$$

where  $\angle_{a_{m(k)n(k)}} g_k$  denotes the phase of  $g_k$  such that  $\angle_{a_{m(k)n(k)}} g_k \in [\pi - a_{m(k)n(k)}, \pi + a_{m(k)n(k)})$  with the real number  $a_{m(k)n(k)}$  defined in Appendix I. Thus,  $\mathbf{x}$  is available for the estimation of the matrix of phases  $\mathbf{Y}$  where element  $(m, n)$  of  $\mathbf{Y}$  is defined as

$$y_{mn} \triangleq \angle_{a_{mn}} h_{mn}. \quad (15)$$

Introducing the vector  $\mathbf{y} = \text{vec}(\mathbf{Y})$  we obtain the following expression for  $\mathbf{x}$

$$\mathbf{x} = \mathbf{S}\mathbf{y} + \boldsymbol{\varphi} + \mathbf{v}, \quad (16)$$

which is valid when the measurement SNR  $\gamma$  is high. In (16), the vector  $\mathbf{v} \triangleq [v_1, \dots, v_K]^T$  is a real-valued additive noise resulting from the additive noise  $\mathbf{w}$ . As shown in Appendix II,  $\mathbf{v}$  can be approximated as  $\mathbf{v} \sim \mathcal{N}(\mathbf{0}, \frac{1}{2\gamma}\mathbf{I})$ .

The linear MMSE estimate of  $\mathbf{y}$  from  $\mathbf{x}$  is obtained as [10]

$$\hat{\mathbf{y}} = \mathbf{x}^T \boldsymbol{\Sigma}_{\mathbf{x}}^{-1} \boldsymbol{\Sigma}_{\mathbf{xy}}, \quad (17)$$

where  $\boldsymbol{\Sigma}_{\mathbf{x}}$  denotes the covariance matrix of  $\mathbf{x}$  and  $\boldsymbol{\Sigma}_{\mathbf{xy}}$  is the covariance matrix of  $\mathbf{x}$  and  $\mathbf{y}$ . We assume that the phases of the channel coefficients are uncorrelated random variables with mean zero. This assumption is a “worst case” as in this case the estimator cannot exploit any correlation between the phases of the sub-channels. We further assume that each element of  $\mathbf{y}$  has variance  $\frac{\pi^2}{3}$  corresponding to the variance of a random variable uniformly distributed on the interval  $[-\pi, \pi)$ . Monte Carlo simulations of the mean square estimation error

TABLE I  
SIMULATION SETTINGS

Setting	Value
$M$	8
$N$	8
$I$	2
$K$	128
$t_k$	$kT$
Sample time $T$	2.54 $\mu\text{s}$
Monte Carlo Runs	100
$\text{rank}(\mathbf{H})$	1
$\gamma \triangleq \sigma_h^2 / \sigma_w^2$	20 dB <sup>†</sup>
$\rho$	35 dB <sup>†</sup>

<sup>†</sup> When no other values are given.

show that this assumption is indeed appropriate. Under these assumptions,  $E(\mathbf{y}) = \mathbf{0}$  and  $\boldsymbol{\Sigma}_{\mathbf{y}} = \frac{\pi^2}{3}\mathbf{I}$ . Hence,  $\boldsymbol{\Sigma}_{\mathbf{xy}}$  reads

$$\boldsymbol{\Sigma}_{\mathbf{xy}} = E(\mathbf{xy}^T) = \mathbf{S}E(\mathbf{yy}^T) = \mathbf{S}\boldsymbol{\Sigma}_{\mathbf{y}} = \frac{\pi^2}{3}\mathbf{S}. \quad (18)$$

where we made use of the fact that  $\boldsymbol{\varphi}$  and  $\mathbf{v}$  have zero-mean and therefore by (16),  $E(\mathbf{x}) = \mathbf{0}$ . Using (16) and (18) the covariance matrix of  $\mathbf{x}$  can be derived as

$$\boldsymbol{\Sigma}_{\mathbf{x}} = \frac{\pi^2}{3}\mathbf{S}\mathbf{S}^T + \boldsymbol{\Sigma}_{\boldsymbol{\varphi}} + \frac{1}{2\gamma}\mathbf{I}, \quad (19)$$

where  $\boldsymbol{\Sigma}_{\boldsymbol{\varphi}}$  is the known covariance matrix of  $\boldsymbol{\varphi}$  defined as

$$[\boldsymbol{\Sigma}_{\boldsymbol{\varphi}}]_{k\ell} \triangleq R_\varphi(t_k - t_\ell). \quad (20)$$

Finally, we propose to use the capacity estimator:

$$\hat{C}_{\text{MMSE}} = C(\hat{\mathbf{H}}\hat{\mathbf{H}}^H), \quad (21)$$

where  $\hat{\mathbf{H}}$  is given in (12) with  $\hat{\mathbf{Z}}$  obtained from (13), and  $\hat{\mathbf{Y}}$  obtained from (17) as  $\hat{\mathbf{Y}} = \text{vec}^{-1}(\hat{\mathbf{y}})$ .

## IV. NUMERICAL RESULTS

We now compare the proposed estimator and the estimators reported in Subsections III-A and III-B by means of Monte Carlo simulations. The simulation settings are reported in Table. I. On the time-scale used in the simulations presented in this contribution, the phase noise process can be modeled as an auto-regressive moving-average (ARMA) process of order (7,6) [5], [7]:

$$\varphi_k = \sum_{p=1}^7 \phi_p \varphi_{k-p} + \sum_{q=1}^6 \theta_q d_{k-q} + d_k, \quad \varphi_k = \varphi(kT) \quad (22)$$

where the driving process  $\{d_k\}$  is a white Gaussian process with sample variance  $\sigma_d^2$  and  $T$  denotes the sample time. The phase noise process was measured using a commercially available sounder as described in [5]. Fig. 3 depicts the sample autocorrelation function of the measured phase noise series together with the autocorrelation function and the coefficients of the fitted ARMA process [5].<sup>2</sup> The sample time  $T = 2.54 \mu\text{s}$  corresponds to twice the duration of a 127-chip long sequence with a chip rate of 100 MHz.

<sup>2</sup>The parameter values reported in [5] differ from the values in Fig. 3 even though the same measurement data was used. This discrepancy is due to an unfortunate misprint in [5].

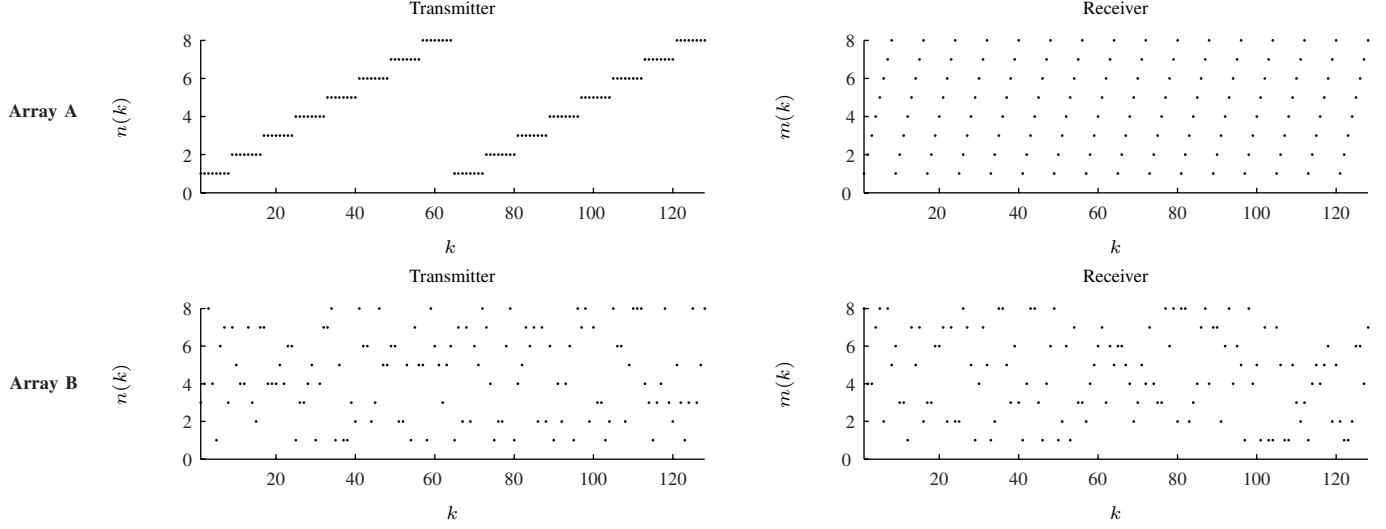


Fig. 2. Spatio-temporal arrays used in the simulations. Array A (upper panels) is the commonly used separable identity array [8]; Array B (lower panels) is a non-separable randomly selected array.

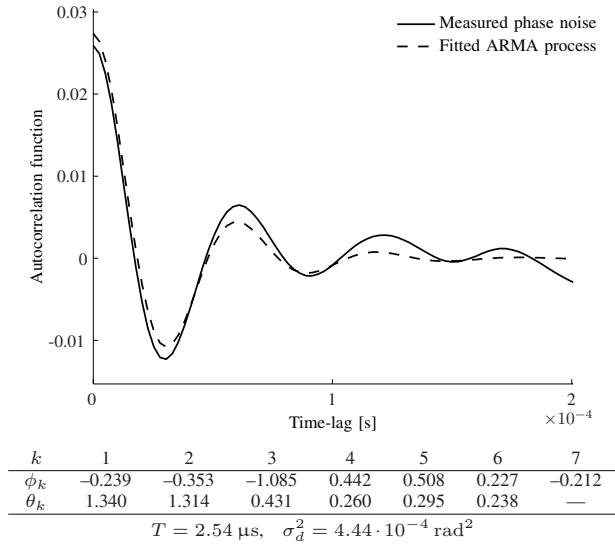


Fig. 3. Sample autocorrelation function (solid line) and autocorrelation function of the ARMA process fitted to the short-term component of phase noise (broken line) [5]. The table reports the parameters of the fitted ARMA process.

In each Monte Carlo run a rank-1 channel matrix  $\mathbf{H}$  (i.e. a key-hole channel) is generated.<sup>3</sup> Then phase noise is generated according to the above model. The average capacity estimates are obtained by averaging over the capacity estimates computed from 100 Monte Carlo runs.

Two different spatio-temporal arrays named Array A and Array B are considered. The two arrays defined by  $m(k)$  and  $n(k)$  are given in Fig. 2. Array A is the commonly used

<sup>3</sup>The impact of phase noise to estimation of capacity is most significant for low rank channels [1], [2]. Thus, despite the fact that a key-hole channels are rare under realistic propagation conditions, the low rank channel is useful for assessing the robustness of a capacity estimator towards phase noise.

identity array [8], [3] and Array B is a non-separable spatio-temporal array optimized for high accuracy and robustness of joint Doppler frequency and direction estimation [8], [3]. Array A is *separable* in the sense that it fulfills the condition [2]

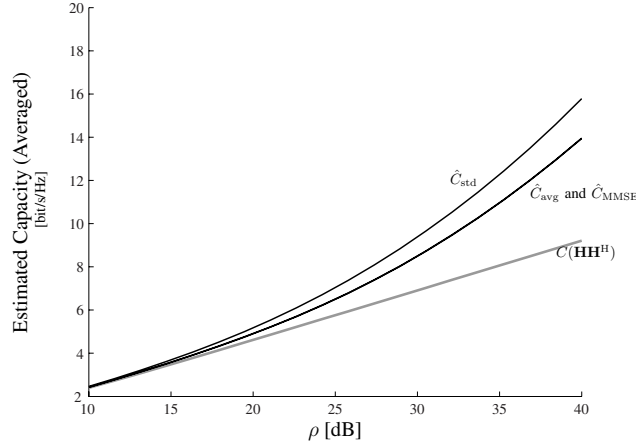
$$t_k = i(k)T_c + [\mathbf{t}_{\text{Tx}}]_{n(k)} + [\mathbf{t}_{\text{Rx}}]_{m(k)}, \quad (23)$$

where  $\mathbf{t}_{\text{Tx}}$  and  $\mathbf{t}_{\text{Rx}}$  are vectors of dimensions  $N$  and  $M$  respectively and  $T_c = MNT$ .

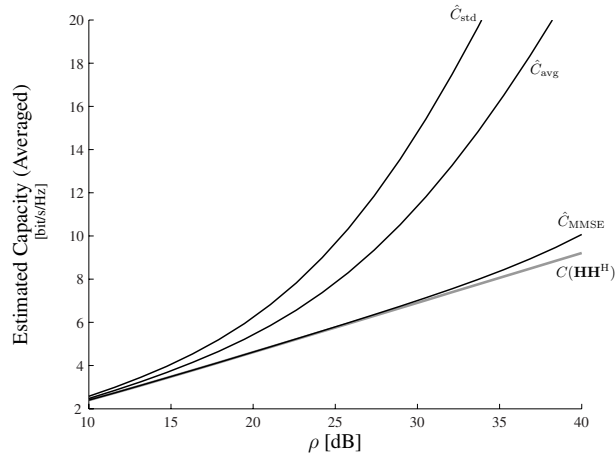
Fig. 4 reports the averaged estimated capacities obtained with the three estimators when Array A and Array B given in Fig. 2 are used. The resulting capacity estimates are plotted as functions of the SNR  $\rho$  as in (7). Fig. 4(a) reports the estimates obtained using Array A. As can be seen from the figure, all three estimators overestimate the capacity for  $\rho > 18$  dB. It can be observed that in this scenario, the estimators  $\hat{C}_{\text{avg}}$  and  $\hat{C}_{\text{MMSE}}$  show the same accuracy, while the standard estimator  $\hat{C}_{\text{std}}$  is less accurate. Fig. 4(b) reports the estimates using Array B. It can be observed that the estimators  $\hat{C}_{\text{std}}$  and  $\hat{C}_{\text{std}}$  perform significantly worse compared to Fig. 4(a) as also reported in [7], while the estimation accuracy of the proposed estimator  $\hat{C}_{\text{MMSE}}$  improves drastically. In Fig. 4(b), the capacity estimates diverge significantly from the true capacity at about 13 dB for estimators  $\hat{C}_{\text{std}}$  and  $\hat{C}_{\text{avg}}$ , and at about 35 dB for the proposed estimator  $\hat{C}_{\text{MMSE}}$ . In practice, MIMO communication systems seldom operate at SNRs as high as 35 dB. Thus, the proposed capacity estimator can be seen to return valid estimates at SNRs considered in practice.

In Fig. 5 the capacity estimates and the mean-square channel estimation error obtained with Array B are reported versus the measurement SNR  $\gamma$  for all three estimators. The mean square error of the channel matrix estimates is computed by averaging the Frobenius norm of the error matrices  $\mathbf{H} - \hat{\mathbf{H}}$  generated in the Monte Carlo runs. It is apparent that the proposed MMSE





(a) Array A



(b) Array B

Fig. 4. Averaged capacity estimates simulated using the settings listed in Fig. 2. Panel (a) reports the results when Array A is used; In panel (b), Array B is used.

estimator yields a capacity estimation error lower than those obtained with the other estimators and that it approaches the exact capacity for  $\gamma$  higher than about 20 dB. Furthermore, it can be seen that the proposed estimator yields a five times lower mean square error than that achieved with the averaging approach (9) for  $\gamma > 20$  dB. This improvement results because the MMSE estimator exploits the known autocorrelation of the phase noise.

## V. CONCLUSIONS

This paper has presented a new estimator for the MIMO channel capacity for the case where the available channel measurements are impaired by both phase noise and additive noise. The proposed estimator relies on separate estimation of the magnitudes and phases of the channel coefficients and exploits knowledge of the phases noise autocorrelation function. This autocorrelation function can be obtained by calibration measurements of the channel sounder. The proposed capacity estimator was compared to conventional methods using two different spatio-temporal arrays. It was found by simulation

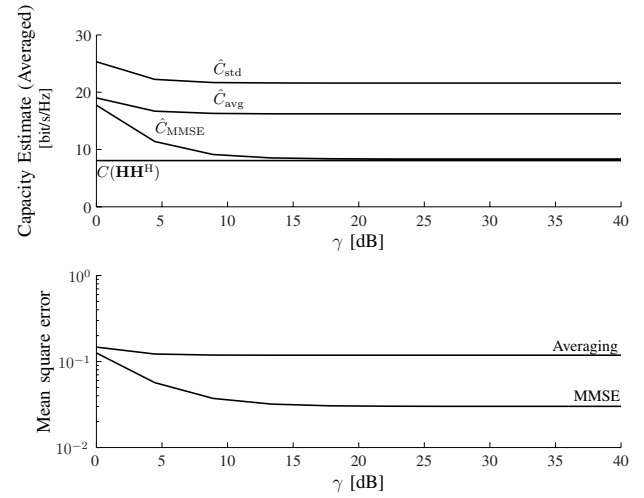


Fig. 5. Averaged capacity estimates and mean square error of the channel matrix estimates versus SNR  $\gamma$ . Array B is used. Upper panel: Estimated capacity with  $\rho = 35$  dB. Lower panel: Mean square error of the channel matrix estimates.

that the accuracy of the proposed capacity estimator is higher when the measurement data is acquired using a non-separable array than when using a separable identity array. Interestingly, the opposite effect applies when the standard capacity estimator is applied: here the separable array leads to the best performance. In conclusion, the simulation results show that the proposed estimator leads to a significant improvement in the estimation of channel capacity from phase-noise impaired measurement data compared to the conventional estimators.

## APPENDIX I

### DEFINITION OF THE ANGLE OPERATOR

The angle of a complex number is a real number that takes a value on an interval of length  $2\pi$ , e.g the interval  $[a - \pi, a + \pi]$  where  $a$  is a real number. We define the mapping

$$\angle_a : \mathbb{C} \rightarrow [a - \pi, a + \pi] \quad \text{s.t.} \quad c = |c| \exp(j\angle_a c). \quad (24)$$

Notice that  $a$  can be any real number. For example it is customary to select  $a = 0$ . However, this causes problems when considering angles between pairs of complex numbers. As an example the numbers  $\exp(j(\pi - \frac{\pi}{8}))$  and  $\exp(j(\pi + \frac{\pi}{8}))$ , differ in angle by  $\frac{\pi}{4}$ , whereas  $\angle_0 \exp(j(\pi - \frac{\pi}{8})) - \angle_0 \exp(j(\pi + \frac{\pi}{8})) = \frac{7\pi}{4}$ .

We assume that the variance of the phase noise components is sufficiently small such that the phases of the samples taken from sub-channel  $(m, n)$  all lie in the interval  $[\angle_0 h_{mn} - \frac{\pi}{2}, \angle_0 h_{mn} + \frac{\pi}{2}]$  with high probability. In this case we can define an angle mapping  $\angle_{a_{mn}}$  where the value of  $a_{mn}$  is defined as the angle of the geometric mean of the set of samples taken from a specific sub-channel  $(m, n)$ :

$$a_{mn} = \angle_0 \left[ \prod_{k \in \mathcal{K}_{mn}} \left( \frac{g_k}{|g_k|} \right)^{\frac{1}{\#\mathcal{K}_{mn}}} \right]. \quad (25)$$

For phase noise processes with sufficiently small sample variance, this per-sub-channel definition of the angle operator

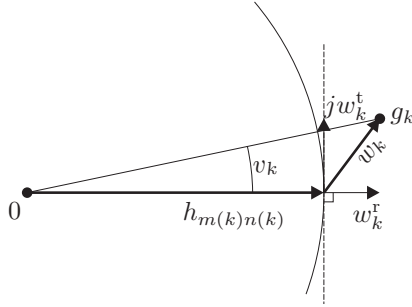


Fig. 6. The noise projection in the complex plane used to compute an approximation of  $\mathbf{v}$  in (16).

enables computation of phase differences between phases of measurements acquired from the same sub-channel by subtraction of the phases.

## APPENDIX II THE ADDITIVE NOISE IN (16)

The  $k$ th noise sample  $v_k$  in (16) denotes the phase contribution due to the additive complex noise sample  $w_k$ . As illustrated in Fig. 6,  $w_k$  can be decomposed into the radial component  $w_k^r$  and the tangential component  $jw_k^t$ . When  $|w_k^r|$  is sufficiently small compared to  $|h_{m(k)n(k)}|$  we can use the approximation  $|h_{m(k)n(k)}|v_k \approx w_k^t$ . Since  $w_k$  is a zero-mean circular symmetric complex Gaussian random variable, the tangential component is Gaussian distributed with variance  $\sigma_w^2/2$ . Then, when the above approximation is valid,  $v_k \sim \mathcal{N}(0, \frac{1}{2\gamma})$ . Thus the covariance matrix of  $\mathbf{v}$  is  $\frac{1}{2\gamma}\mathbf{I}$ .

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